On the Capacity of Causal Cognitive Interference Channel With Delay

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Abstract—In this paper, we introduce the Causal Cognitive Interference Channel With Delay (CC-IFC-WD) in which the cognitive user transmission can depend on L future received symbols as well as the past ones. Taking the effect of the link delays into account, CC-IFC-WD fills the gap between the genie-aided and causal cognitive radio channels. We study three special cases: 1) Classical CC-IFC ($L = 0$), 2) CC-IFC without delay ($L = 1$) and 3) CC-IFC with a block length delay ($L = n$). In each case, we obtain an inner bound on the capacity region. Our coding schemes make use of cooperative strategy by generalized block Markov superposition coding, collaborative strategy by rate splitting, and Gel'fand-Pinsker coding in order to pre-cancel part of the interference. Moreover, instantaneous relaying and non-causal partial Decode-and-Forward strategies are employed in the second and third cases, respectively. The derived regions under special conditions, reduce to several previously known results. Moreover, we show that the coding strategy which we use to derive achievable rate region for the classical CC-IFC achieves capacity for a special case of this channel. Furthermore, we extend our achievable rate regions to Gaussian case. Providing a numerical example for Gaussian CC-IFC-WD, we investigate the rate gain of the cognitive link for different delay values.

I. INTRODUCTION

Cognitive Interference Channel (C-IFC) refers to a two-user interference channel in which the cognitive user (secondary user) has the ability to obtain the message being transmitted by the other user (primary user), either in a non-causal or causal manner. C-IFC was first introduced in [1], where for the non-causal C-IFC an achievable rate region is derived by combining the Gel'fand-Pinsker (GP) binning [2] and a well known simultaneous super-position coding scheme (rate splitting) applied to the Interference Channel (IFC) [3]. For the non-causal C-IFC, where the cognitive user has non-causal full or partial knowledge of the other user's transmitted message, several achievable rate regions and capacity results in some special cases have been established [4]-[7].

In the Causal C-IFC (CC-IFC), the cognitive user can exploit knowledge of the primary user's message from the causally received signals (information overheard by the feedback link from the channel and not sent back from the receivers). Due to the complex nature of the problem, CC-IFC which is a more realistic and appropriate model for practical applications than the non-causal C-IFC, has been far less investigated compared with the latter [8]. In [1], achievable rate regions for the CC-IFC that consist of the non-cooperative causal transmission protocols have

been characterized. An improved rate region for CC-IFC employing a cooperative coding strategy based on the block Markov superposition coding (full Decode-and-Forward (DF) [9]) and GP coding was derived in [10]. A more general model in which both transmitters are causally cognitive was proposed in [11], called Interference Channel with Generalized Feedback (IFC-GF). Different achievable rate regions for IFC-GF were obtained in [11]-[13], combining the methods of rate splitting, block Markov superposition coding, and GP binning.

In this paper, we define the Causal Cognitive Interference Channel With Delay (CC-IFC-WD) as an IFC where one of the transmitters can causally overhear the channel and its transmission can depend on the L future received symbols as well as the past ones. This can equivalently be seen as the classical CC-IFC with $-L$ unit delay on the cognitive user's received signal (or on the link between the transmitters). This channel model fits the wireless networks where the transmitters are close. Moreover, CC-IFC-WD is a middle point between the unrealistic genie-aided (non-causal) C-IFC and complex CC-IFC. In fact, a simple strategy such as Instantaneous Relaying (IR) by itself could be beneficial, as the case in the Relay With Delay (RWD) channel [14]. Different upper and lower bounds and some capacity results have been derived for RWD in [14]-[16], where the lower bounds are achieved based on the combination of cooperative strategies such as: full or partial DF, IR (for $L > 0$), and non-causal DF (for $L = n$). It has been shown that the capacity of the RWD is strictly larger than the classic relay channel [14].

In this paper, after introducing the general CC-IFC-WD, we focus on three special cases: 1) $L = 0$ which corresponds to the classical CC-IFC, 2) CC-IFC without delay ($L = 1$) where current received symbol (at the cognitive user) could also be utilized and 3) CC-IFC with a block length delay ($L = n$), in which cognitive user knows its entire received sequence non-causally. In each case, we obtain new inner bound on the capacity region (achievable rate region) for the general discrete memoryless case. Our coding schemes benefit the cooperative strategy by generalized block Markov coding (partial DF [9]) and superposition coding, collaborative strategy by rate splitting and GP coding in order to mitigate part of the interference. For the first case (classic CC-IFC), we use a different strategy compared to the previous results. We use partial DF instead of full DF. Therefore our achievable region improves that of [10]. Moreover, since common message should be decoded in both receivers, binning against the common message provides no improvement. Therefore, we use GP binning to pre-cancel the part of the private message. A similar result has been concluded in [17] for the Cognitive Z-IFC. In the second and third cases, besides the approach we adopt for the first case, IR and

non-causal partial DF strategies are employed, respectively. The derived achievable rate regions, under special conditions, reduce to several previously known rate regions, such as the ones in [3], [14]. Moreover, we derive the capacity region for a special case of the CC-IFC-WD, where achievablity follows from our derived region. Furthermore, we consider Gaussian CC-IFC-WD and extend the achievable rate regions for $L = 0$, $L = 1$ and $L = n$, to the Gaussian case. Providing a numerical example for Gaussian CC-IFC-WD, we investigate the rate gain of the cognitive link for different delay values. Thus, we compare the strategies which are used for our coding schemes and show that IR and non-causal DF improve the rate region noticeably.

The rest of the paper is organized as follows. Section II introduces the general CC-IFC-WD channel model and the notations. In Section III, we consider three different scenarios and derive new inner bound on the capacity region for each scenario. Capacity region for a special case of the CC-IFC-WD is derived in Section IV. In Section V, Gaussian CC-IFC-WD is investigated.

II. CHANNEL MODELS AND PRELIMINARIES

Throughout the paper, upper case letters (e.g. X) are used to denote random variables (RVs) and lower case letters (e.g. x) show their realizations. The probability mass function (p.m.f) of a random variable (RV) X with alphabet set $\mathcal{X}$, is denoted by $p_X(x)$, where occasionally subscript X is omitted. $|\mathcal{X}|$ denotes the cardinality of a finite discrete set $\mathcal{X}$. $A_\epsilon^n(X, Y)$ specifies the set of ϵ-strongly, jointly typical sequences of length n, abbreviated by A_ϵ^n if it is clear. The notation X_i^j indicates a sequence of RVs $(X_i, X_{i+1}, ..., X_j)$, where we use X^j instead of X_1^j, for brevity.

Consider the CC-IFC-WD in Fig.1, which is denoted by $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_2, y_3, y_4|x_1, x_2), \mathcal{Y}_2 \times \mathcal{Y}_3 \times \mathcal{Y}_4)$, where $X_1 \in \mathcal{X}$ and $X_2 \in \mathcal{X}_2$ are inputs of Transmitter 1 (Tx1) and Transmitter 2 (Tx2), respectively, $Y_2 \in \mathcal{Y}_2$ is the secondary user output, $Y_3 \in \mathcal{Y}_3$ and $Y_4 \in \mathcal{Y}_4$ are channel outputs at the Receiver 1 (Rx1) and Receiver 2 (Rx2), respectively, $p(y_2, y_3, y_4|x_1, x_2)$ is the channel transition probability distribution. In n channel uses, each Txu sends a message m_u to the Rxu where $u \in \{1, 2\}$.

Definition 1: A $(2^{nR_1}, 2^{nR_2}, n)$ code for the CC-IFC-WD consists of (i) two message sets $\mathcal{M}_1 = \{1, ..., 2^{nR_1}\}$ and $\mathcal{M}_2 = \{1, ..., 2^{nR_2}\}$ for the primary and secondary users, respectively, (ii) an encoding function at the primary user $f_1 : \mathcal{M}_1 \mapsto \mathcal{X}_1^n$, (iii) a set of encoding functions at the secondary user $x_{2,i} = f_{2,i}(m_2, y_2^{i-1+L})$, for $1 \leq i \leq n$ and $m_2 \in \mathcal{M}_2$, (iv) two decoding functions at Rx1 and Rx2, $g_1 : \mathcal{Y}_3^n \mapsto \mathcal{M}_1$ and $g_2 : \mathcal{Y}_4^n \mapsto \mathcal{M}_2$. We assume that the channel is memoryless. Thus, for $m_1 \in \mathcal{M}_1$ and $m_2 \in \mathcal{M}_2$, the joint p.m.f of $\mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y}_2 \times \mathcal{Y}_3 \times \mathcal{Y}_4$ is given by

$$p(m_1, m_2, x_1, x_2, y_2, y_3, y_4) = p(m_1)p(m_2) \prod_{i=1}^{n} p(x_{1,i}|m_1)$$
$$\times p(x_{2,i}|m_2, y_2^{i-1+L})p(y_{2,i}|x_{1,i})p(y_{3,i}, y_{4,i}|x_{1,i}, x_{2,i})$$
(1)

where we avoid instantaneous feedback from X_2 to Y_2, which delay may cause. The probability of error for this code is defined as $P_e = max\{P_{e,1}, P_{e,2}\}$, where for $u \in \{1, 2\}$ we have:

$$P_{e,u} = \frac{1}{2^{n(R_1+R_2)}} \sum_{m_1, m_2} P(g_u(Y_{u+2}^n) \neq m_u|(m_1, m_2) \text{ sent})$$

Definition 2: A rate pair (R_1, R_2) is achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes with $P_e \to 0$ as $n \to \infty$. The capacity region $\mathcal{C}_L$, is closure of set of all achievable rates.
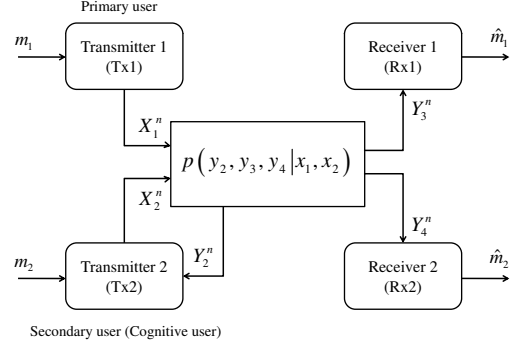

Fig. 1. Causal Cognitive Interference Channel With Delay (CC-IFC-WD)

III. DISCRETE MEMORYLESS CC-IFC-WD

In this section, we consider the discrete memoryless CC-IFC-WD and concentrate on three special cases: 1) Classical CC-IFC ($L = 0$), 2) CC-IFC without delay ($L = 1$) where current received symbol (at the cognitive user) can be utilized too and 3) CC-IFC with a block length delay ($L = n$), in which cognitive user knows its entire received sequence non-causally. For all setups, new inner bounds on the capacity region are derived. We utilize a coding scheme which is based on combining generalized block Markov superposition coding, rate splitting and GP binning for part of the interference. Furthermore, we apply IR in the second setup and non-causal partial DF in the last case. The outline of the proofs are presented.

A. Classical CC-IFC ($L = 0$)

We present a new achievable rate region for this setup. Consider auxiliary RVs $T_c, T_p, U_{1c}, U_{1p}, V_{1c}, V_{1p}, U_{2c}, U_{2p}$ and a time sharing RV Q defined on arbitrary finite sets $\mathcal{T}_c, \mathcal{T}_p, \mathcal{U}_{1c}, \mathcal{U}_{1p}, \mathcal{V}_{1c}, \mathcal{V}_{1p}, \mathcal{U}_{2c}, \mathcal{U}_{2p}$ and $\mathcal{Q}$, respectively. Let $Z_1 = (Q, T_c, T_p, U_{1c}, U_{1p}, V_{1c}, V_{1p}, U_{2c}, U_{2p}, X_1, X_2, Y_2, Y_3, Y_4)$, and $\mathcal{P}_1$ denotes the set of all joint p.m.fs $p(.)$ on Z_1 that can be factored in the form of

$$p(z_1) = p(q)p(t_c|q)p(t_p|t_c, q)p(u_{1c}|t_c, q)p(u_{1p}|u_{1c}, t_p, t_c, q)$$
$$\times p(v_{1c}|t_c, q)p(v_{1p}|v_{1c}, t_p, t_c, q)p(x_1|v_{1p}, v_{1c}, u_{1p}, u_{1c}, t_p, t_c, q)$$
$$\times p(u_{2c}, u_{2p}|t_p, t_c, q)p(x_2|u_{2c}, u_{2p}, t_p, t_c, q)p(y_2, y_3, y_4|x_1, x_2)$$
(2)

Let $\mathcal{R}_1(Z_1)$ denote set of all nonnegative rate pairs (R_1, R_2) where $R_1 = R_{10d} + R_{10n} + R_{11d} + R_{11n}$ and $R_2 = R_{20} + R_{22}$, such that there exists nonnegative (L_{20}, L_{22}) satisfying:

$$L_{20} \geq I(U_{2c}; T_p|T_cQ) \doteq I_1 \qquad (3)$$
$$L_{22} \geq I(U_{2p}; T_p|T_cQ) \doteq I_2 \qquad (4)$$
$$L_{20} + L_{22} \geq I(U_{2c}; U_{2p}|T_cQ) + I(U_{2c}U_{2p}; T_p|T_cQ) \doteq I_3 \qquad (5)$$
$$R_{11n} \leq I(V_{1p}; Y_3|U_{2c}V_{1c}U_{1p}U_{1c}T_pT_cQ) \doteq I_4 \qquad (6)$$
$$R_{10d} + R_{10n} + R_{11d} + R_{11n} + L_{20} + R_{20}$$
$$\leq I(U_{2c}V_{1p}V_{1c}U_{1p}U_{1c}T_pT_c; Y_3|Q) \doteq I_5 \qquad (7)$$
$$R_{10n} + R_{11d} + R_{11n}$$
$$\leq I(V_{1p}V_{1c}U_{1p}T_p; Y_3U_{2c}|U_{1c}T_cQ) \doteq I_6 \qquad (8)$$
$$R_{11d} + R_{11n} \leq I(V_{1p}U_{1p}T_p; Y_3U_{2c}|V_{1c}U_{1c}T_cQ) \doteq I_7 \qquad (9)$$
$$R_{11d} + R_{11n} + L_{20} + R_{20}$$
$$\leq I(U_{2c}V_{1p}U_{1p}T_p; Y_3|V_{1c}U_{1c}T_cQ) \doteq I_8 \qquad (10)$$

$$R_{10n} + R_{11n} \leq I(V_{1c}V_{1p}; Y_3|U_{2c}U_{1p}U_{1c}T_pT_cQ) \doteq I_9 \quad (11)$$

$$R_{11n} + L_{20} + R_{20} \leq I(V_{1p}U_{2c}; Y_3|V_{1c}U_{1p}U_{1c}T_pT_cQ) \doteq I_{10} (12)$$

$$R_{10n} + R_{11d} + R_{11n} + L_{20} + R_{20}$$
$$\leq I(U_{2c}V_{1p}V_{1c}U_{1p}T_p; Y_3|U_{1c}T_cQ) \doteq I_{11} \quad (13)$$

$$R_{10n} + R_{11n} + L_{20} + R_{20}$$
$$\leq I(U_{2c}V_{1p}V_{1c}; Y_3|U_{1p}U_{1c}T_pT_cQ) \doteq I_{12} \quad (14)$$

$$L_{20} + R_{20} \leq I(U_{2c}; Y_4U_{2p}|V_{1c}U_{1c}T_cQ) \doteq I_{13} \quad (15)$$

$$L_{22} + R_{22} \leq I(U_{2p}; Y_4U_{2c}|V_{1c}U_{1c}T_cQ) \doteq I_{14} \quad (16)$$

$$R_{10d} + R_{10n} + L_{20} + R_{20} + L_{22} + R_{22}$$
$$\leq I(U_{2c}U_{2p}V_{1c}U_{1c}T_c; Y_4|Q) \doteq I_{15} \quad (17)$$

$$R_{10n} + L_{20} + R_{20} \leq I(U_{2c}V_{1c}; Y_4U_{2p}|U_{1c}T_cQ) \doteq I_{16} (18)$$

$$R_{10n} + L_{22} + R_{22} \leq I(U_{2p}V_{1c}; Y_4U_{2c}|U_{1c}T_cQ) \doteq I_{17} (19)$$

$$R_{10n} + L_{20} + R_{20} + L_{22} + R_{22}$$
$$\leq I(U_{2c}U_{2p}V_{1c}; Y_4|U_{1c}T_cQ) \doteq I_{18} \quad (20)$$

$$L_{20} + R_{20} + L_{22} + R_{22}$$
$$\leq I(U_{2c}U_{2p}; Y_4|V_{1c}U_{1c}T_cQ) \doteq I_{19} \quad (21)$$

$$R_{11d} \leq I(U_{1p}; Y_2|U_{2c}U_{2p}U_{1c}T_pT_cQ) \doteq I_{20} \quad (22)$$

$$R_{10d} + R_{11d} \leq I(U_{1c}U_{1p}; Y_2|U_{2c}U_{2p}T_pT_cQ) \doteq I_{21} \quad (23)$$

Theorem 1: For any $p(.) \in \mathcal{P}_1$ the region $\mathcal{R}_1(Z_1)$ is an achievable rate region for the discrete memoryless classical CC-IFC (CC-IFC-WD with $L = 0$), i.e., $\bigcup_{Z_1 \in \mathcal{P}_1} \mathcal{R}_1(Z_1) \subseteq \mathcal{C}_0$.

Remark 1: Consider the case where cognitive user can not overhear the channel, i.e. $Y_2 = 0$. If we set $T_c = T_p = U_{1c} = U_{1p} = \emptyset$, $L_{20} = L_{22} = R_{10d} = R_{11d} = 0$ and $V_{1p} = X_1$, the rate region reduces to Han-Kobayashi (HK) region [3].

Remark 2: If we omit reciever 2, i.e. $Y_4 = 0$, and cognitive user does not have any message to transmit, i.e. $R_2 = 0$, the model reduces to the relay channel. By setting $T_c = T_p = U_{1p} = V_{1p} = U_{2p} = \emptyset$, $L_{20} = L_{22} = R_{11n} = R_{11d} = R_2 = 0$ and $U_{2c} = X_2$, the rate region reduces to the partial DF rate for the relay channel [9], which includes the capacity regions of the degraded [9] and semi-deterministic relay channels [18].

Outline of the Proof: We propose the following random coding scheme, which contains regular generalized block Markov superposition coding, rate splitting and GP coding in the encoding part. For decoding at the receivers we utilize backward decoding. Messages of the primary and cognitive users are split into four and two parts, respectively, i.e.: $m_1 = (m_{10d}, m_{10n}, m_{11d}, m_{11n})$ and $m_2 = (m_{20}, m_{22})$, where subscript d (or n) refers to the part of the primary user's message which can (or can not) be decoded by the cognitive user. Moreover, (m_{10}, m_{20}) and (m_{11}, m_{22}) are common and private messages as in the HK scheme [3]. m_{10d} should be decoded at Rx2 (besides its intended receiver), therefore binning against m_{10d} at the cognitive transmitter provides no improvement. Hence, cognitive user cooperatively with the primary user sends m_{10d}, while uses GP binning method against m_{11d} to mitigate the effect of this known interference at Rx2. Now, consider a block Markov encoding scheme with B blocks of transmission, each of n symbols.

Codebook Generation: Let q^n be a random sequence according to $\prod_{i=1}^{n} p(q_i)$. Generate $2^{nR_{10d}}$ independent identically distributed (i.i.d) t_c^n sequences, each with probability $\prod_{i=1}^{n} p(t_{c,i}|q_i)$. Index them as $t_c^n(m'_{10d})$ where $m'_{10d} \in [1, 2^{nR_{10d}}]$. For each

$t_c^n(m'_{10d})$, generate $2^{nR_{11d}}$ i.i.d t_p^n sequences, $2^{nR_{10d}}$ i.i.d u_{1c}^n sequences and $2^{nR_{10n}}$ i.i.d v_{1c}^n sequences, according to $\prod_{i=1}^{n} p(t_{p,i}|t_{c,i}, q_i)$, $\prod_{i=1}^{n} p(u_{1c,i}|t_{c,i}, q_i)$ and $\prod_{i=1}^{n} p(v_{1c,i}|t_{c,i}, q_i)$, respectively. Index them as $t_p^n(m'_{11d}, m'_{10d})$, $u_{1c}^n(m_{10d}, m'_{10d})$ and $v_{1c}^n(m_{10n}, m'_{10d})$ where $m'_{11d} \in [1, 2^{nR_{11d}}]$, $m_{10d} \in [1, 2^{nR_{10d}}]$ and $m_{10n} \in [1, 2^{nR_{10n}}]$. For each $(u_{1c}^n(m_{10d}, m'_{10d}), t_p^n(m'_{11d}, m'_{10d}), t_c^n(m'_{10d}))$, generate $2^{nR_{11d}}$ i.i.d u_{1p}^n sequences, according to $\prod_{i=1}^{n} p(u_{1p,i}|u_{1c,i}, t_{p,i}, t_{c,i}, q_i)$. Index them as $u_{1p}^n(m_{11d}, m_{10d}, m'_{11d}, m'_{10d})$ where $m_{11d} \in [1, 2^{nR_{11d}}]$. For each $(v_{1c}^n(m_{10n}, m'_{10d}), t_p^n(m'_{11d}, m'_{10d}), t_c^n(m'_{10d}))$, generate $2^{nR_{11n}}$ i.i.d v_{1p}^n sequences, according to $\prod_{i=1}^{n} p(v_{1p,i}|v_{1c,i}, t_{p,i}, t_{c,i}, q_i)$. Index them as $v_{1p}^n(m_{11n}, m_{10n}, m'_{11d}, m'_{10d})$ where $m_{11n} \in [1, 2^{nR_{11n}}]$. From the p.m.f in (2), compute the marginals $p(u_{2c}|t_c, q)$ and $p(u_{2p}|t_c, q)$ (drop the dependence on t_p). For each $t_c^n(m'_{10d})$, generate $2^{n(R_{20}+L_{20})}$ i.i.d u_{2c}^n sequences, each with probability $\prod_{i=1}^{n} p(u_{2c,i}|t_{c,i}, q_i)$. Index them as $u_{2c}^n([m_{20}, l_{20}], m'_{10d})$, where $m_{20} \in [1, 2^{nR_{20}}]$ and $l_{20} \in [1, 2^{nL_{20}}]$. For each $t_c^n(m'_{10d})$, generate $2^{n(R_{22}+L_{22})}$ i.i.d u_{2p}^n sequences, according to $\prod_{i=1}^{n} p(u_{2p,i}|t_{c,i}, q_i)$. Index them as $u_{2p}^n([m_{22}, l_{22}], m'_{10d})$ with $m_{22} \in [1, 2^{nR_{22}}]$, $l_{22} \in [1, 2^{nL_{22}}]$.

Encoding *(at the beginning of block b):*

Primary User: In order to transmit the message $m_{1,b} = (m_{10d,b}, m_{10n,b}, m_{11d,b}, m_{11n,b})$, Tx1 picks codewords $v_{1p}^n(m_{11n,b}, m_{10n,b}, m_{11d,b-1}, m_{10d,b-1}), v_{1c}^n(m_{10n,b}, m_{10d,b-1})$, $u_{1p}^n(m_{11d,b}, m_{10d,b}, m_{11d,b-1}, m_{10d,b-1}), u_{1c}^n(m_{10d,b}, m_{10d,b-1})$, $t_p^n(m_{11d,b-1}, m_{10d,b-1}), t_c^n(m_{10d,b-1})$. Then, sends x_1^n generated according to $\prod_{i=1}^{n} p(x_{1,i}|v_{1p,i}, v_{1c,i}, u_{1p,i}, u_{1c,i}, t_{p,i}, t_{c,i}, q_i)$. In the first block cooperative information is: $(m_{11d,b-1}, m_{10d,b-1}) = (m_{11d,0}, m_{10d,0}) = (0, 0)$ and in the last block, a previously known message $(m_{11d,B}, m_{10d,B}) = (1, 1)$ is transmitted.

Cognitive User: Tx2 at the beginning of block b, knows $\tilde{m}_{10d,b-1}$ and $\tilde{m}_{11d,b-1}$, which are estimates of the parts of the common and private messages sent by Tx1 in the previous block and can be decoded by the cognitive user. In order to send $m_{2,b} = (m_{20,b}, m_{22,b})$, encoder 2 knowing codewords $t_p^n(\tilde{m}_{11d,b-1}, \tilde{m}_{10d,b-1})$ and $t_c^n(\tilde{m}_{10d,b-1})$, seeks an index pair $(l_{20,b}, l_{22,b})$ such that

$$(u_{2c}^n([m_{20,b}, l_{20,b}], \tilde{m}_{10d,b-1}), u_{2p}^n([m_{22,b}, l_{22,b}], \tilde{m}_{10d,b-1}),$$
$$t_p^n(\tilde{m}_{11d,b-1}, \tilde{m}_{10d,b-1}), t_c^n(\tilde{m}_{10d,b-1}), q^n) \in A_\epsilon^n \quad (24)$$

If there is more than one such index pair, pick the smallest. If there are no such codewords, declare an error. There exist such indices $l_{20,b}$ and $l_{22,b}$ with enough high probability, if n is sufficiently large and (3)-(5) hold. Then, it sends x_2^n generated according to $\prod_{i=1}^{n} p(x_{2,i}|u_{2p,i}, u_{2c,i}, t_{p,i}, t_{c,i}, q_i)$.

Decoding: *Cognitive User:* Tx2 at the end of block b, wants to correctly recover $(m_{11d,b}, m_{10d,b})$. Hence, it looks for a unique pair $(\tilde{m}_{11d,b}, \tilde{m}_{10d,b})$ such that

$$(y_2^n(b), u_{1p}^n(\tilde{m}_{11d,b}, \tilde{m}_{10d,b}, m_{11d,b-1}, m_{10d,b-1}),$$
$$u_{1c}^n(\tilde{m}_{10d,b}, m_{10d,b-1}), u_{2c}^n([m_{20,b}, l_{20,b}], m_{10d,b-1}),$$

$$u_{2p}^n([m_{22,b}, l_{22,b}], m_{10d,b-1}), t_p^n(m_{11d,b-1}, m_{10d,b-1}), \quad (25)$$
$$t_c^n(m_{10d,b-1}), q^n) \in A_\epsilon^n(Y_2, U_{2c}, U_{2p}, U_{1p}, U_{1c}, T_p, T_c, Q)$$

This step can be done with small enough probability of error, for sufficiently large n if (22)-(23) hold.

Backward decoding is used at the receivers, hence they start decoding after all B blocks are received. *Rx1:* In block b, Rx1 looks for a unique quadruple $(m_{11n,b}, m_{10n,b}, m_{11d,b-1}, m_{10d,b-1})$ and some pair $(m_{20,b}, l_{20,b})$ such that

$$(y_3^n(b), u_{2c}^n([m_{20,b}, l_{20,b}], m_{10d,b-1}), v_{1c}^n(m_{10n,b}, m_{10d,b-1}),$$
$$v_{1p}^n(m_{11n,b}, m_{10n,b}, m_{11d,b-1}, m_{10d,b-1}), \quad (26)$$
$$u_{1p}^n(m_{11d,b}, m_{10d,b}, m_{11d,b-1}, m_{10d,b-1}), t_c^n(m_{10d,b-1}),$$
$$u_{1c}^n(m_{10d,b}, m_{10d,b-1}), t_p^n(m_{11d,b-1}, m_{10d,b-1}), q^n) \in A_\epsilon^n$$

where $(m_{11d,b}, m_{10d,b})$ were decoded in the previous step. Here, for large enough n, the probability of error can be made sufficiently small if (6)-(14) hold.

Rx2: In block b, Rx2 finds a unique pair $(m_{20,b}, m_{22,b})$ and some quadruple $(l_{20,b}, l_{22,b}, m_{10n,b}, m_{10d,b-1})$ such that

$$(u_{2c}^n([m_{20,b}, l_{20,b}], m_{10d,b-1}), u_{2p}^n([m_{22,b}, l_{22,b}], m_{10d,b-1}),$$
$$v_{1c}^n(m_{10n,b}, m_{10d,b-1}), u_{1c}^n(m_{10d,b}, m_{10d,b-1}), t_c^n(m_{10d,b-1}),$$
$$q^n, y_4^n(b)) \in A_\epsilon^n(Y_4, U_{2c}, U_{2p}, V_{1c}, U_{1c}, T_c, Q) \quad (27)$$

where $m_{10d,b}$ was decoded in the previous step. With arbitrary high probability, no error occurs in Rx2 if n is large enough and (15)-(21) hold. ∎

Now to understand the shape of the achievable region, we give a compact expression for $\mathcal{R}_1(Z_1)$ which is easier to compute.

Corollary 1: The region $\mathcal{R}_1(Z_1)$, after Fourier-Motzkin elimination, can be expressed as:

$$R_1 \le min\Big(min(I_{21} + I_2' + I_{16}, I_{21} + I_{12}, I_5) - I_1,$$
$$I_{21} + min(I_2' + I_{17} - I_2, I_9)\Big)$$
$$R_2 \le min\Big(I_{19}, I_{14} + min(I_{10}, I_{13})\Big) - I_1'$$
$$R_1 + R_2 \le min\Big(I_{14} + I_5, I_{15} + min(I_7, I_8 - I_1),$$
$$I_{21} + I_{17} + min(I_{10}, I_2' + I_{13}),$$
$$I_{21} + I_{14} + min(I_{12}, I_2' + I_{16}, I_{10} + I_{17} - I_2),$$
$$I_2' + min(I_{21} + I_{18}, I_{20} + I_{15})\Big) - I_1'$$
$$2R_1 + R_2 \le min\Big(I_2' + I_{15} + I_3', I_{21} + 2I_4 + I_{17} + I_{16},$$
$$I_4 + I_{17} + min(I_{21} + I_{12}, I_5)\Big) + I_{21} - I_1'$$
$$R_1 + 2R_2 \le min\Big(I_{21} + I_{10} + I_{14} + min(I_{14} + I_{16}, I_{18}),$$
$$I_{14} + I_{15} + min(I_{20} + I_{10}, I_8)\Big) - 2I_1'$$
$$2R_1 + 2R_2 \le min\Big(I_4 + min(I_{14} + I_{11}, I_{17} + I_8),$$
$$I_{10} + I_{14} + I_3'\Big) + I_{21} + I_{15} - 2I_1'$$
$$2R_1 + 3R_2 \le I_{21} + I_{10} + 2I_{14} + I_{11} + I_{15} - 3I_1'$$
$$3R_1 + 2R_2 \le 2I_{21} + 2I_4 + I_{11} + I_{17} + I_{15} - 2I_1'$$

subject to $I_1 \le min(I_8, I_{10}, I_{12}, I_{13}, I_{16})$ and $I_2 \le I_{17}$, where $\{I_i, 1 \le i \le 21\}$ were defined in (3)-(23), and $I_1' \doteq max(I_1 + I_2, I_3)$, $I_2' \doteq min(I_{10} - I_1, I_4)$, and $I_3' \doteq min(I_6, I_{11} - I_1)$.

B. CC-IFC without delay ($L = 1$)

In this case, cognitive user could utilize the current received symbol as well as the past ones in order to cooperate with the primary user or reduce the interference effect. In addition to the scheme used in Theorem 1, IR is employed to achieve higher rates using this additional information. Consider auxiliary RVs $T_c, T_p, U_{1c}, U_{1p}, V_{1c}, V_{1p}, U_{2c}, U_{2p}, V_2$ and Q defined on arbitrary finite sets $\mathcal{T}_c, \mathcal{T}_p, \mathcal{U}_{1c}, \mathcal{U}_{1p}, \mathcal{V}_{1c}, \mathcal{V}_{1p}, \mathcal{U}_{2c}, \mathcal{U}_{2p}, \mathcal{V}_2$ and $\mathcal{Q}$, respectively. Let $Z_2 = (Z_1, V_2)$, and $\mathcal{P}_2$ be the set of all joint p.m.fs $p(.)$ on Z_2 that can be factored in the form of

$$p(z_2) = p(q)p(t_c|q)p(t_p|t_c, q)p(u_{1c}|t_c, q)p(u_{1p}|u_{1c}, t_p, t_c, q)$$
$$\times p(v_{1c}|t_c, q)p(v_{1p}|v_{1c}, t_p, t_c, q)p(x_1|v_{1p}, v_{1c}, u_{1p}, u_{1c}, t_p, t_c, q)$$
$$\times p(u_{2c}, u_{2p}|t_p, t_c, q)p(v_2|u_{2c}, u_{2p}, t_p, t_c, q)p(x_2|v_2, y_2, q) \quad (28)$$

In fact $x_2 = f_2'(v_2, y_2, q)$, where $f_2'(.)$ is an arbitrary deterministic function. Let $\mathcal{R}_2(Z_2)$ be the set of all nonnegative rate pairs (R_1, R_2) where $R_1 = R_{10d} + R_{10n} + R_{11d} + R_{11n}$ and $R_2 = R_{20} + R_{22}$, such that there exists nonnegative real (L_{20}, L_{22}) which satisfy (3)-(23).

Theorem 2: For any $p(.) \in \mathcal{P}_2$ the region $\mathcal{R}_2(Z_2)$ is achievable for the discrete memoryless CC-IFC without delay (CC-IFC-WD with $L = 1$), i.e., $\bigcup_{Z_2 \in \mathcal{P}_2} \mathcal{R}_2(Z_2) \subseteq \mathcal{C}_1$.

Proof: The achievability proof follows by combining the scheme used in Theorem 1 and IR. Encoding and decoding follow the same lines as Theorem 1, except that during the codebook generation at the cognitive user (Tx2), v_2^n is generated according to $\prod_{i=1}^n p(v_{2,i}|u_{2p,i}, u_{2c,i}, t_{p,i}, t_{c,i}, q_i)$, and in the encoding session, Tx2 at time i and upon receiving $y_{2,i}$, sends $x_{2,i} = f_{2,i}'(v_{2,i}, y_{2,i}, q_i)$. ∎

Remark 3: If we assume that $\mathcal{V}_2$ has extended alphabet of size $|\mathcal{X}_2|^{|\mathcal{Y}_2|}$ (all mappings from $\mathcal{Y}_2$ to $\mathcal{X}_2$), this scheme is analogous to *Shannon's strategy* of cancelling the causally known interference [19], [20].

Remark 4: This scheme is feasible for any $L \ge 1$. Moreover, $f_2'(.)$ can be extended to $x_{2,i} = f_{2,i}'(v_{2,i}, y_{2,i}, ..., y_{2,i+L-1}, q_i)$.

Remark 5: Nullifying $T_c = T_p = U_{1p} = V_{1p} = U_{2p}$, setting $R_2 = L_{20} = L_{22} = R_{11n} = R_{11d} = 0$ and $U_{2c} = V_2$, the region reduces to the partial DF rate for RWD [14, Theorem 2.5].

C. CC-IFC with a block length delay ($L = n$)

In this part, we investigate CC-IFC with a block length delay ($L = n$). This means that cognitive user knows its entire received sequence non-causally. We derive an achievable rate region using a coding scheme based on combining non-causal partial DF, rate splitting and GP binning against part of the interference. Consider auxiliary RVs $U_{1c}, U_{1p}, V_{1c}, V_{1p}, U_{2c}, U_{2p}$ and a time sharing RV Q defined on arbitrary finite sets $\mathcal{U}_{1c}, \mathcal{U}_{1p}, \mathcal{V}_{1c}, \mathcal{V}_{1p}, \mathcal{U}_{2c}, \mathcal{U}_{2p}$ and $\mathcal{Q}$, respectively. Let $Z_3 = (Q, U_{1c}, U_{1p}, V_{1c}, V_{1p}, U_{2c}, U_{2p}, X_1, X_2, Y_2, Y_3, Y_4)$, and $\mathcal{P}_3$ denote the set of all joint p.m.fs $p(.)$ on Z_3 that can be factored in the form of (2) with $(t_p, t_c) = (u_{1p}, u_{1c})$. Let $\mathcal{R}_3(Z_3)$ be the set of all nonnegative rate pairs (R_1, R_2) where $R_1 = R_{10d} + R_{10n} + R_{11d} + R_{11n}$ and $R_2 = R_{20} + R_{22}$ such that there exists nonnegative real (L_{20}, L_{22}) which satisfy (3)-(21) with $(T_p, T_c) = (U_{1p}, U_{1c})$ and:

$$R_{11d} \le I(U_{1p}; Y_2|U_{1c}Q) \quad (29)$$
$$R_{10d} + R_{11d} \le I(U_{1c}U_{1p}; Y_2|Q) \quad (30)$$

Theorem 3: For any $p(.) \in \mathcal{P}_3$ the region $\mathcal{R}_3(Z_3)$ is achievable for the discrete memoryless CC-IFC with a block length delay (CC-IFC-WD with $L = n$), i.e., $\bigcup_{Z_3 \in \mathcal{P}_3} \mathcal{R}_3(Z_3) \subseteq \mathcal{C}_n$.

Proof: Proof is similar to Theorem 1, except that there is no dependence on previous block messages. Hence, simultaneous joint decoding is used instead of backward decoding. ∎

IV. CAPACITY OF DEGRADED CLASSICAL CC-IFC

In this section, we investigate classical CC-IFC (CC-IFC-WD with $L = 0$), with joint p.m.f p^*, given by (1) with $L = 0$. Using the achievable region in Theorem 1, we find the capacity region for a special case. We define degraded classical CC-IFC as a classical CC-IFC where degradedness condition for the Tx1-Rx1 pair with cognitive user as a relay holds for every p^*:

$$p(y_3|x_1, x_2, y_2) = p(y_3|x_2, y_2) \qquad (31)$$

i.e., $X_1 \rightarrow (X_2, Y_2) \rightarrow Y_3$ forms a Markov chain. Next, we impose the following strong interference conditions:

$$I(X_1; Y_3) \leq I(X_1; Y_4) \qquad (32)$$
$$I(X_2; Y_4|X_1) \leq I(X_2; Y_3|X_1) \qquad (33)$$

In fact, under these conditions interfering signals at Rx1 and Rx2 are strong enough to decode both messages.

Theorem 4: The capacity region of the degraded classical CC-IFC with joint p.m.f p^*, satisfying (32) and (33), is given by

$$\mathcal{C}_0^* = \bigcup_{p(t)p(x_1|t)p(x_2|t)} \left\{ (R_1, R_2) : R_1 \geq 0, R_2 \geq 0 \right.$$
$$R_1 \leq (X_1; Y_2|X_2 T) \qquad (34)$$
$$R_2 \leq I(X_2; Y_4|X_1 T)$$
$$\left. R_1 + R_2 \leq min\{I(X_1 X_2; Y_3), I(X_1 X_2; Y_4)\} \right\}$$

Proof: Achievability: For this part, we use the region $\mathcal{R}_1$ in Theorem 1 (or Corollary 1) and we ignore time sharing RV Q. Let, $T_p = U_{1p} = V_{1p} = U_{2p} = \emptyset$ and $R_{22} = R_{11n} = R_{11d} = 0$, which cross out the private parts of both messages, making the messages common to both receivers. Furthermore, assume that cognitive user fully decode the message of the primary user (m_1). Hence, set $R_{10n} = 0$ and $V_{1c} = \emptyset$. To omit the GP coding, we set $L_{20} = L_{22} = 0$. Redefining $T_c = T$, $U_{2c} = X_2$, $U_{1c} = X_1$ and applying condition (33) complete the proof for the achievability.

<u>Converse:</u> Consider a $(2^{nR_1}, 2^{nR_2}, n)$ code with $P_e \rightarrow 0$. For $1 \leq i \leq n$, define the RV $T_i = Y_2^{i-1}$. Noting joint p.m.f p^*, we remark that $X_1 \rightarrow T \rightarrow X_2$ forms a Markov chain. First, we provide a useful lemma which we need in the proof of converse.

Lemma 1: If (32) and (33) hold for all distribution p^*, then

$$I(X_1^n; Y_3^n) \leq I(X_1^n; Y_4^n) \qquad (35)$$
$$I(X_2^n; Y_4^n|X_1^n) \leq I(X_2^n; Y_3^n|X_1^n) \qquad (36)$$

Proof: Proof follows the same lines as in [5, Lemma 5]. ∎

Noting the independence of the messages and utilizing Fano's inequality for the first bound, we have:

$$nR_1 - n\delta_{1n} \leq I(M_1; Y_3^n|M_2) \overset{(a)}{\leq} I(M_1; Y_3^n Y_2^n|M_2)$$
$$\overset{(b)}{=} \sum_{i=1}^{n} I(M_1; Y_{3,i} Y_{2,i}|Y_3^{i-1} Y_2^{i-1} M_2 X_{2,i}) \quad (37)$$
$$\overset{(c)}{\leq} \sum_{i=1}^{n} I(M_1 X_{1,i} Y_3^{i-1}; Y_{3,i} Y_{2,i}|X_{2,i}, T_i)$$

$$\overset{(d)}{=} \sum_{i=1}^{n} I(X_{1,i}; Y_{3,i} Y_{2,i}|X_{2,i}, T_i)$$

where (a) and (c) due to the fact that mutual information is non-negative, (b) obtains from the chain rule and the fact that $X_{2,i}$ is a deterministic functions of M_2 and Y_2^{i-1} and (d) because channel is memoryless. Using standard time-sharing argument and condition (31), we have

$$R_1 - \delta_{1n} \leq I(X_1; Y_3, Y_2|X_2, T) = I(X_1; Y_2|X_2, T)$$

Similarly, applying Fano's inequality, we bound R_2 as:

$$nR_2 - n\delta_{2n} \leq I(M_2; Y_4^n|M_1) \leq I(M_1; Y_4^n Y_2^n|M_1)$$
$$\overset{(a)}{=} \sum_{i=1}^{n} I(M_2; Y_{4,i}, Y_{2,i}|Y_4^{i-1}, Y_2^{i-1}, M_1)$$
$$\overset{(b)}{\leq} \sum_{i=1}^{n} I(M_2, X_{2,i}; Y_{4,i}, Y_{2,i}|Y_4^{i-1}, T_i, M_1, X_{1,i})$$
$$\overset{(c)}{\leq} \sum_{i=1}^{n} I(X_{2,i}; Y_{4,i}|X_{1,i}, T_i)$$

where (a) is based on the chain rule, (b) since $X_{1,i}$ is a deterministic functions of M_1 and mutual information is non-negative and (c) because channel is memoryless with joint p.m.f p^*.

Now, we utilize Fano's inequality to bound $R_1 + R_2$ as:

$$n(R_1 + R_2) - n\delta_{3n} \leq I(M_1; Y_3^n) + I(M_2; Y_4^n|M_1)$$
$$\overset{(a)}{\leq} I(X_1^n; Y_3^n) + I(M_2; Y_4^n|M_1)$$
$$\overset{(b)}{\leq} I(X_1^n; Y_3^n) + I(M_2, X_2^n; Y_4^n|M_1, X_1^n)$$
$$\overset{(c)}{\leq} I(X_1^n; Y_3^n) + I(X_2^n; Y_4^n|X_1^n) \qquad (38)$$
$$\overset{(d)}{\leq} I(X_1^n; Y_3^n) + I(X_2^n; Y_3^n|X_1^n)$$
$$= I(X_1^n X_2^n; Y_3^n) \leq \sum_{i=1}^{n} I(X_{1,i} X_{2,i}; Y_{3,i})$$

where (a) and (b) follow from the non-negativity of the mutual information and the deterministic relation between X_1^n and M_1, (c) follows from fact that conditioning does not increase the entropy and the conditional independence between Y_4 and (M_1, M_2) given (X_1, X_2), and (d) holds due to (36).

Finally, utilizing Fano's inequality, (38-c) and condition (35), the last bound can be shown. Using standard time-sharing argument for these bounds, completes the proof. ∎

V. GAUSSIAN CC-IFC-WD

We consider Gaussian CC-IFC-WD and extend the achievable rate regions $\mathcal{R}_1(Z_1)$, $\mathcal{R}_2(Z_2)$ and $\mathcal{R}_3(Z_3)$ derived for the discrete memoryless classical CC-IFC ($L = 0$), CC-IFC without delay ($L = 1$) and CC-IFC with a block length delay ($L = n$), respectively, to the Gaussian case.

Gaussian CC-IFC-WD at time $i = 1, ..., n$, can be modeled as

$$Y_{2,i} = h_{21} X_{1,i} + Z_{2,i}$$
$$Y_{3,i} = h_{31} X_{1,i} + h_{32} X_{2,i} + Z_{3,i} \qquad (39)$$
$$Y_{4,i} = h_{41} X_{1,i} + h_{42} X_{2,i} + Z_{4,i}$$

where, h_{21}, h_{31}, h_{32}, h_{41} and h_{42} are known channel gains. $X_{1,i}$ and $X_{2,i}$ are input signals with average power constraints P_1 and

P_2, respectively. $Z_{2,i}, Z_{3,i}$ and $Z_{4,i}$ are i.i.d and independent zero mean Gaussian noise components with powers N_2, N_3 and N_4, respectively. Note that, at the secondary user we have a set of encoding functions $x_{2,i} = f_{2,i}(m_2, y_2^{i-1+L})$ for $1 \leq i \leq n$ and $m_2 \in \mathcal{M}_2$.

First, we consider Gaussian classical CC-IFC ($L = 0$). Using standard arguments, region $\mathcal{R}_1$ in Theorem 1 (or Corollary 1) can be extended to the discrete-time Gaussian memoryless case with continuous alphabets ($\mathcal{R}_1^*$). Hence, it is sufficient to evaluate the (3)-(23) with an appropriate choice of input distribution. We constrain all the inputs to be Gaussian and set the time sharing RV $Q = \emptyset$. For certain $\{0 \leq \beta_r \leq 1, r \in \{1, 2, 3, 4\}\}$, $\{0 \leq \beta_s' \leq 1, s \in \{1, 2\}\}$ and $\{0 \leq \gamma_t \leq 1, t \in \{1, 2, 3\}\}$ with $\beta_1' + \beta_1 + \beta_2' + \beta_2 + \beta_3 + \beta_4 \leq 1$ and $\gamma_1 + \gamma_2 + \gamma_3 \leq 1$, consider the following mapping (MAP_1) for the generated codebook in Theorem 1 with respect to the p.m.f (2), which contains rate splitting, generalized block Markov superposition and GP coding:

$$T_c \sim \mathcal{N}(0, \beta_4 P_1), T_p' \sim \mathcal{N}(0, \beta_3 P_1), U_{1c}' \sim \mathcal{N}(0, \beta_2 P_1) \quad (40)$$

$$V_{1c}' \sim \mathcal{N}(0, \beta_1 P_1), U_{1p}' \sim \mathcal{N}(0, \beta_2' P_1), V_{1p}' \sim \mathcal{N}(0, \beta_1' P_1) \quad (41)$$

$$T_p = T_p' + T_c, U_{1c} = U_{1c}' + T_c, V_{1c} = V_{1c}' + T_c \quad (42)$$

$$U_{1p} = U_{1p}' + U_{1c}' + T_p' + T_c, V_{1p} = V_{1p}' + V_{1c}' + T_p' + T_c \quad (43)$$

$$X_1 = V_{1p}' + V_{1c}' + U_{1p}' + U_{1c}' + T_p' + T_c \quad (44)$$

$$U_{2c}' \sim \mathcal{N}(0, \gamma_1 P_2), U_{2p}' \sim \mathcal{N}(0, \gamma_2 P_2) \quad (45)$$

$$S_1 = h_{41} T_p' \ , \ S_2 = h_{41} T_p' + h_{42} U_{2c}' \quad (46)$$

$$U_{2c} = U_{2c}' + \alpha_1 S_1 \ , \ U_{2p} = U_{2p}' + \alpha_2 S_2 \quad (47)$$

$$X_2 = U_{2p}' + U_{2c}' + \sqrt{\frac{\gamma_3 P_2}{\beta_4 P_1}} T_c \quad (48)$$

where we have $\alpha_1 \doteq \frac{h_{42} \gamma_1 P_2}{A + h_{42}^2 \gamma_2 P_2}$ and $\alpha_2 \doteq \frac{h_{42} \gamma_2 P_2}{A}$ wherein $A = N_4 + h_{41}^2 (\beta_1' + \beta_1 + \beta_2' + \beta_2) P_1 + h_{42}^2 \gamma_1 P_2 + \left(h_{41} \sqrt{\beta_4 P_1} + h_{42} \sqrt{\gamma_3 P_2}\right)^2$. In fact, optimal values for α_1, α_2, S_1 and S_2 (used for GP coding) can be found by optimizing the rate region. However, this method is cumbersome and we use the modified version of Costa's dirty paper coding (DPC) results [21].

Next, we investigate Gaussian CC-IFC without delay ($L = 1$), i.e. $X_2 = f_2(m_2, Y_2^i)$. Considering Theorem 2, $v_{2,i}$ is generated according to $p(v_{2,i} | u_{2p,i}, u_{2c,i}, t_{p,i}, t_{c,i}, q_i)$, and $x_{2,i} = f_{2,i}'(v_{2,i}, y_{2,i}, q_i)$. In order to obtain the Gaussian counterpart of $\mathcal{R}_2$, namely $\mathcal{R}_2^*$, appropriate mapping (MAP_2), consists of (40)-(47), $V_2 = U_{2p}' + U_{2c}' + \sqrt{\frac{\gamma_3 P_2}{\beta_4 P_1}} T_c$ and $X_2 = h(\beta Y_2 + (1-\beta) V_2)$ where $0 \leq \beta \leq 1$ and h is a normalizing parameter.

Finally, we consider the Gaussian CC-IFC with a block length delay ($L = n$). We can use $\mathcal{R}_1^*$ to obtain the Gaussian counterpart of $\mathcal{R}_3$, namely $\mathcal{R}_3^*$. The only difference is that according to Theorem 3, there is no dependence on the previous block messages. Therefore, we can set $T_c = U_{1c}$ and $T_p = U_{1p}$, or equivalently $\beta_2' = \beta_2 = 0$ in (MAP_1) to obtain MAP_3.

Fig. 2 compares $\mathcal{R}_1^*, \mathcal{R}_2^*, \mathcal{R}_3^*$ with HK region in [3], where the overheard information is neglected. For $L = 0$, rate improvement over HK region can be seen, especially when cognitive link is good enough ($h_{21} = 4$). Due to IR, even when $h_{21} = 1$, $\mathcal{R}_2^*$ outperforms both $\mathcal{R}_1^*$ and HK region, significantly. Considering $\mathcal{R}_2^*$ and $\mathcal{R}_3^*$, it is seen that when R_2 is small, IR can achieve higher rates than non-causal DF. However, when cognitive user sends at higher rates, condition of the cognitive link determines better strategy. Note that, using a coding scheme based on the
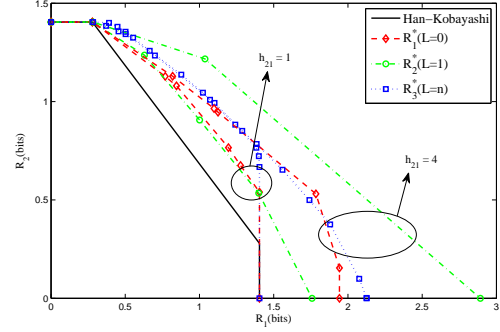

Fig. 2. Comparison between $\mathcal{R}_1^*, \mathcal{R}_2^*, \mathcal{R}_3^*$ and HK region [3]. $P_1 = P_2 = 6$, $h_{31} = h_{42} = 1$, $h_{32} = h_{41} = \sqrt{0.55}$ and $N_2 = N_3 = N_4 = 1$.

combination of IR and non-causal DF, convex hull of $\mathcal{R}_2^*$ and $\mathcal{R}_3^*$ is achievable for CC-IFC with $L = n$.

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